Traitement du signal

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1 Analyse spectrale

$x(t) = \sum_{n=0}^{\infty} A_n \cdot \cos(2\pi f_n t + \varphi_n)$

1.1 Discret

$fm = m\Delta f$

$\sum_{m=0}^{\infty} a(m\Delta f)\Delta f \cos(2\pi m\Delta ft + \varphi m)$

$c_2(t) = A_2 \cos(2\pi f_2 t + \varphi_2) \approx \int_0^{\infty} \frac{1}{\Delta f} \sqcap_{\Delta f} (f - f_2) A_2 \cos(2\pi ft + \varphi_2) df \approx \cos(2\pi f_2 t + \varphi_2) \int_0^{\infty} \frac{1}{\Delta f} \sqcap_{\Delta f} (f - f_2) df$

$[\ldots]$

$x(t) = \inf_{-\infty}^{\infty} X(f) e^{i2\pi ft} df$

$e^{i2\pi ft} = cos(2\pi ft) + i \sin(2\pi ft)$

$Xf() = |X(f)| e^{i\varphi(f)}$

$x(t) = \inf_{-\infty}^{\infty} |X(f)| e^{i\varphi(t)} e^{i2\pi ft} df$

$x(t) = \inf_{-\infty}^{\infty} |X(f)| e^{i(2\pi ft + \varphi(t))} df$

$e^{i(2\pi ft + \varphi(t))} = cos(2\pi ft + \varphi(t)) + isin(2\pi ft + varphi(t))$

$\boxed{\varphi(f) = arg(X(f))}$

$X(f) = \frac{1}{2} a(f) e^{i\varphi(f)} \in \mathbb{C}$

$a(f) = 2|X(f)| \in \mathbb{R}^+$

$\varphi(f) = arg(X(f)) \in \mathbb{R}$

$a(-f) = a(f)$

$\varphi(-f) = -\varphi(-f)$

Exercice :

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1. Calculer le spectre de $x(t) = A \sin(2\pi f_1 t + \varphi_1)$
2. En Déduire la densité de l'amplitude spectrale a(f)

1.

$TF[e^{2i\pi f_1 t}] = ?$

$TF[\delta(t)] = \int\limits_{-\infty}^{\infty} \delta(t) e^{2i\pi f t} dt = \int\limits_{-\infty}^{\infty} \delta(t) e^{-2i\pi f 0} dt = \delta(t) \int\limits_{-\infty}^{\infty} e^{-2i\pi f 0} dt = 1$

$TF^{-1}[\delta(f)] = \int\limits_{-\infty}^{\infty} \delta(f) e^{2i\pi f t} df = 1$

$TF^{-1}[\delta(f - f_1)] = \int\limits_{-\infty}^{\infty} \delta(f - f_1) e^{2i\pi f t} df = e^{2i\pi f_1 t} \int\limits_{-\infty}^{\infty} \delta(f - f_1) df = e^{2i\pi f_1 t}$

$TF[e^{2i\pi f_1 t}] = \delta(f - f_1)$

$sin(2\pi f_1 t + \varphi_1) = \frac{e^{i(2\pi f_1 t + \varphi_1)} - e^{i(2\pi f_1 t + \varphi_1)}}{2i} = \frac{1}{2i}[e^{i\varphi_1} TF[e^{i2\pi f_1 t}] - e^{-i\varphi_1} TF[e^{i2\pi f_1 t}]] = \frac{1}{2i}[TF[e^{i\varphi_1} e^{i2\pi f_1 t}] - TF[e^{-i\varphi_1} e^{-i2\pi f_1 t}]]$

$X(f) = \frac{e^{i\varphi_1}}{2i} \delta[f - f1] - \frac{e^{-i\varphi_1}}{2i} \delta(f + f1)$

$|X(f)| = \left| \frac{e^{i\varphi_1}}{2i} \delta[f - f1] - \frac{e^{-i\varphi_1}}{2i} \delta(f + f1) \right| = |\frac{e^{i\varphi_1}}{2e^{i\frac{\pi}{2}}} \delta(f - f_1)| + |\frac{e^{-i\varphi_1}}{2e^{i\frac{\pi}{2}}} \delta(f + f_1)| = \frac{1}{2}[\delta(f - f_1) + \delta(f + f_1)]$