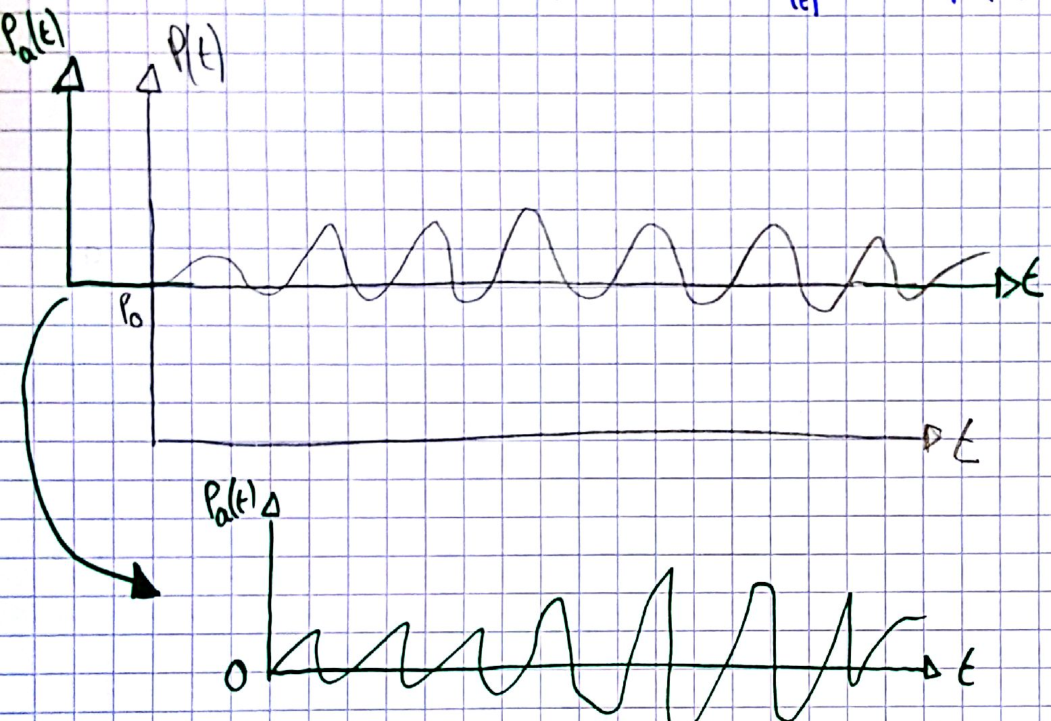
TAN - $\boxed{01/09/15}$

① Intro analyse spectrale

$P_0 \simeq 10^5$ Pa
100 000 N/m²
1 kgf/cm²
} Pression atmosphérique

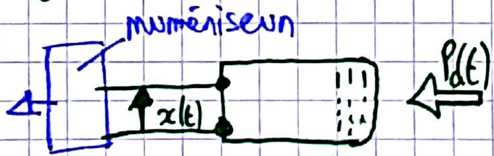
Pa → pression acoustique

$P_{(t)} = P_0 + Pa(t)$ → pression totale



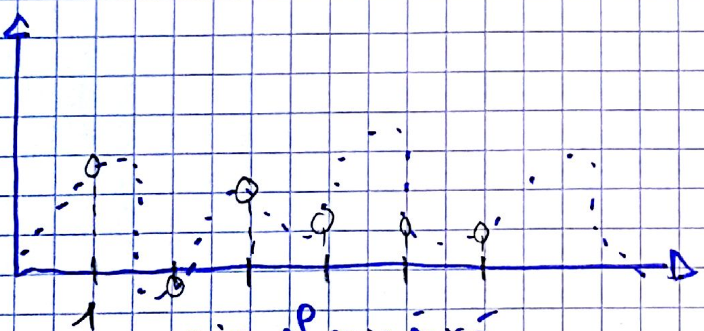
$x(t) = \alpha \times P_a(t)$

- $x(t)$ ↓ tension électrique (en volt)
- α ↓ sensibilité micro
- $P_a(t)$ ↓ en Pa

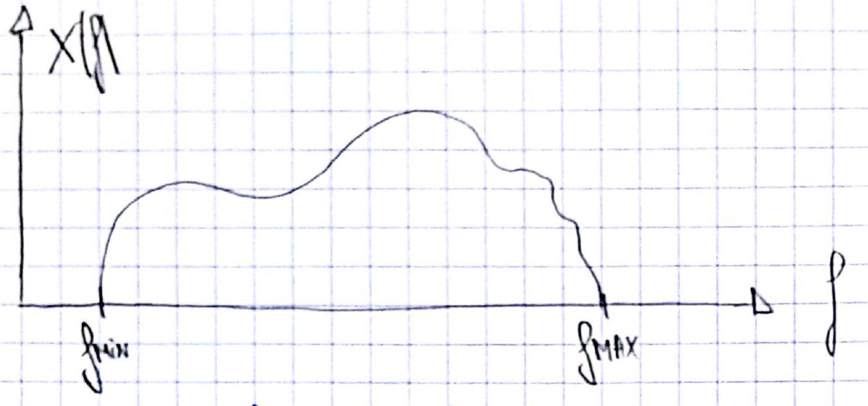


Capteur Analogique

x(t) est l'analogie de Pa en tension électrique



signal numérisé



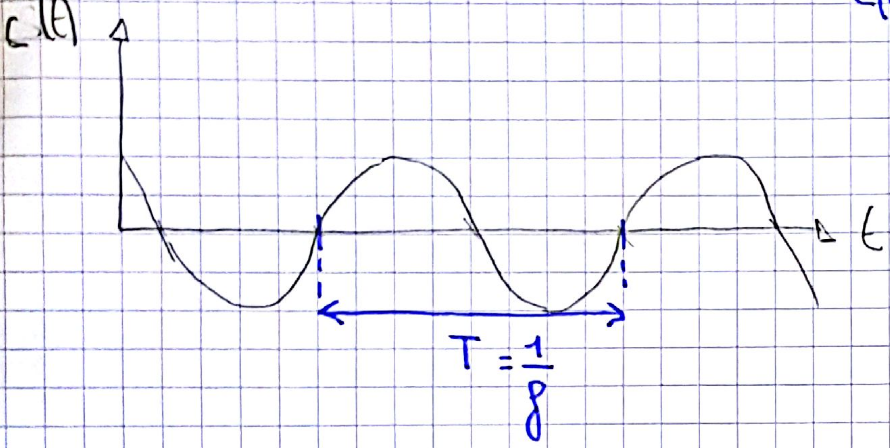
$$f_e \geq 2 \times f_{MAX}$$

6 Sinusoïde

fréquence

$$c(t) = A \cdot \cos(2\pi f \cdot t + \varphi)$$

phase à l'origine

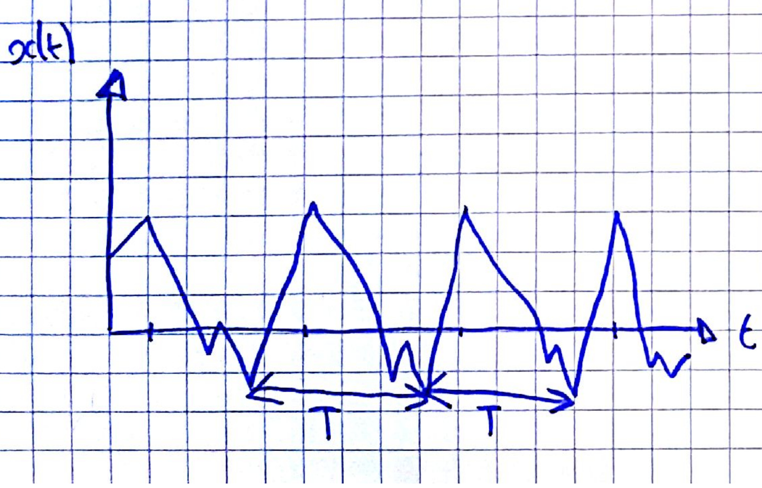


$$y(t) = H[x(t)]$$

système linéaire

$$\boxed{\begin{array}{c} H[a(t) + b(t)] \\ = H[a(t)] + H[b(t)] \\ H[\lambda \cdot x(t)] = \lambda \cdot H[x(t)] \end{array}}$$

Stationnaire : $H[x(t-3)] = y(t-3)$



$$x(t+T) = x(t)$$

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Fourier

Série de Fourier :

$$x(t) = \sum_{k=0}^{+\infty} A_k \cdot \cos(2\pi \underbrace{k f_1}_{f_k} t + \varphi_k)$$

$$\boxed{f_k = k \cdot f_1}\quad \begin{cases} f_1 = \text{fréquence fondamental} \\ k = \text{ordre de l'harmonique} \\ k \cdot f_1 = \text{fréquence harmonique} \end{cases}$$

$$H[x(t)] = H\left[\sum_{k=0}^{+\infty} A_k \cdot \cos(2\pi \cdot k \cdot f_1 t + \varphi_k)\right]$$

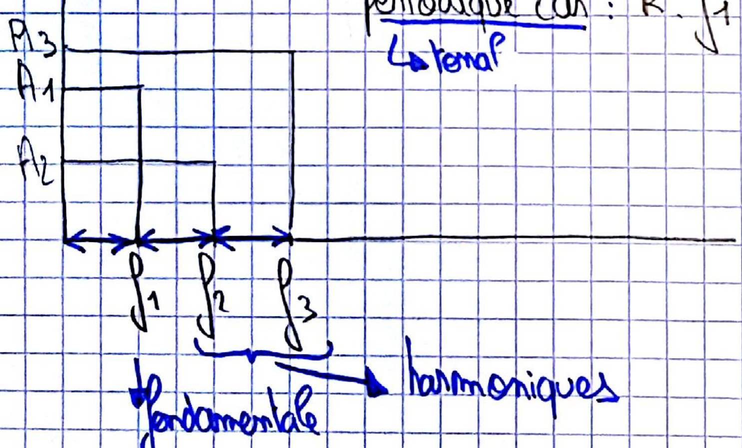
$$= \sum_{k=0}^{+\infty} H\left[A_k \cdot \cos(2\pi \cdot k \cdot f_1 + \varphi_k)\right]$$

$$= \sum_{k=0}^{+\infty} A_k \, H\left[\cos(2\pi \cdot k \cdot f_1 + \varphi_k)\right]$$

$$y(t) = \sum_{k=0}^{+\infty} A'_k \cos(2\pi \cdot k f_1 + \varphi'_k)$$

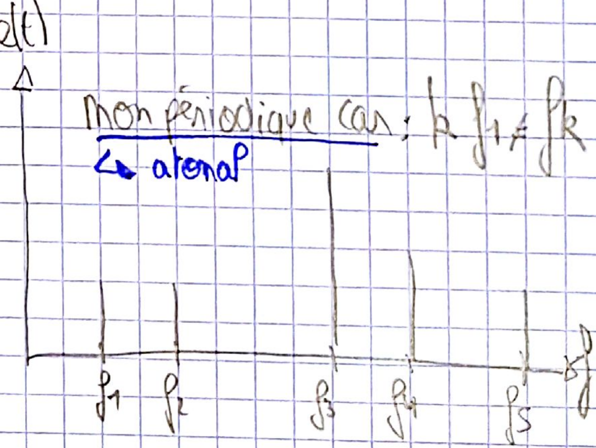
périodique cas : $k \cdot f_1 = f_k$

↳ tonal



non périodique cas : $k \cdot f_1 \neq f_k$

↳ atonal



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SON (suite)

- **élasticité d'un gaz :**

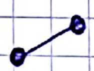
$$\chi_Q = \frac{1}{\gamma P_0} \xrightarrow{} \text{dépend de } \lambda$$

$$\boxed{\gamma = 1 + \frac{2}{\lambda}}$$

$$\chi = - \frac{\frac{dV}{V}}{dP}$$

$\frac{dV}{V}$ → variation relative de volume

dP → variation de pression

nb degré de liberté λ	
3	• He
3+2 = 5	 O_2 N_2
3+3 = 6	 NH_3

Ain: bi-atomique ⇒ $\lambda = 5$ $\quad \gamma = 1 + \frac{2}{5} = 1 + \frac{4}{10} = 1 + 0,4 = 1,4$

- **masse volumique d'un gaz**

$$\rho = \frac{m}{V}$$

$$PV = NRT$$

P : pression, V : volume, N : mole, T : température

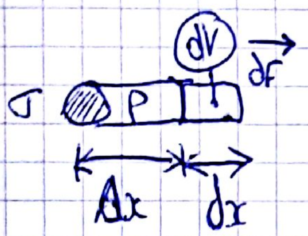
$$\rho = \frac{N.M}{V} \quad \text{avec} \quad PV = NRT$$

$$V = \frac{NRT}{P}$$

$$\text{donc } \rho = \frac{N.M}{\frac{NRT}{P}} = \frac{P.M}{RT}$$

$$\boxed{\rho = \frac{P.M}{R.T}}$$

$$\Delta x = \overbrace{\Delta x . \sigma . \rho}^{m = V\rho}$$



$$dV = dx . \sigma$$

$$V = \sigma . \Delta x$$

$$dF = dP . \sigma$$

dF et dx

$$\chi = \frac{\sigma}{\Delta x} . \frac{1}{k} \qquad \boxed{k = \frac{\sigma}{\Delta x . \chi}}$$

$$\boxed{\frac{\partial^2 p}{\partial x^2} - \chi \rho . \frac{\partial^2 p}{\partial t^2} = 0}$$

$$p(x,t) = A.\cos(2\pi f t + \varphi) \qquad \omega = 2\pi f$$

$$= A.\cos(\omega t + \varphi)$$

$$p(x, t - \tau) = A.\cos(\omega.(t - \tau) + \varphi_2)$$

$$\tau = \frac{\ell}{v}$$

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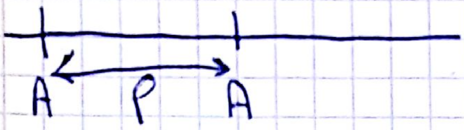

$$p(0,t) = A\cdot\cos(2\pi f t + \varphi_0)$$
$$A\cdot\cos(\omega t + \varphi_0)$$

$$p(x,t) = p(0,t-\tau)$$
$$= A\cdot\cos(\omega(t-\tau) + \varphi)$$
$$A\cdot\cos(\omega t - \omega\tau + \varphi) \qquad \tau = \frac{x}{v}\left(\frac{d}{v}\right)$$
$$A\cdot\cos\left(\omega t - \omega\frac{x}{v} + \varphi\right)$$

$$p(x,t) = A\cos\left(\omega t - \frac{\omega}{v}\cdot x + \varphi_0\right)$$

$$\frac{\partial p}{\partial t} = A\cdot\frac{\partial\left(\omega t - \frac{\omega}{v}\cdot x + \varphi_0\right)}{\partial t}\cdot\left(-\sin\left(\omega t - \frac{\omega}{v}x + \varphi_0\right)\right)$$
$$= -A\cdot\omega\sin\left(\omega t - \frac{\omega}{v}\cdot x + \varphi_0\right)$$

$$\frac{\partial^2 p}{\partial t^2} = -A\cdot\omega\cdot\frac{\partial}{\partial t}\left[\sin\left(\underbrace{\omega t - \frac{\omega}{v}x + \varphi_0}_{f(t)}\right)\right] \qquad [\sin(f(t))]' = f'(t)\cos(f(t))$$

$$= -A\cdot\omega\cdot\omega\cdot\cos\left(\omega t - \frac{\omega}{v}x + \varphi_0\right)$$

$$\boxed{\frac{\partial^2 p}{\partial t^2} = -\omega^2\cdot p}$$

$$\frac{\partial p}{\partial x} = \frac{\omega}{v}A\sin\left(\omega t - \frac{\omega}{v}x + \varphi_0\right)$$
$$\frac{\partial^2 p}{\partial x^2} = -\left(\frac{\omega}{v}\right)^2\boxed{A\cos\left(\omega t - \frac{\omega}{v}x + \varphi_0\right)} = \boxed{-\left(\frac{\omega}{v}\right)^2\cdot p}$$

$$-\left(\frac{\omega}{V}\right)^2 \rho - X.\rho.(-\omega^2 \rho) = 0$$

$$-\frac{1}{V^2} - X.\rho.(-1) = 0$$

$$V^2 = \frac{1}{X.\rho} \qquad V = \sqrt{\frac{1}{\sqrt{X.\rho}}}$$

$$(R_0)\ \boxed{\rho = \frac{P.M}{R.T}} \qquad \boxed{X = \frac{1}{\gamma.P}}$$

$$V = \frac{1}{\sqrt{X.\frac{PM}{RT}}} = \frac{1}{\sqrt{\frac{1}{\gamma P}.\frac{PM}{RT}}} = \frac{1}{\sqrt{\frac{M}{\gamma RT}}} = \sqrt{\frac{\gamma RT}{M}}$$

<u>Mair</u> : 79% N_2 + 21% O_2 = 28,84 g/mol

N ≈ 14g/mole N_2 = 28

O ≈ 16 g/mole O_2 = 32

M = 0,02884 kg/mol

γ = 1,4

T ≈ 273°K ⟶ 331,9 m/s

R = 8,314 J/K

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Impédance acoustique

Z_A $\begin{cases} u \text{ vitesse de l'air} \\ p \text{ pression acoustique} \end{cases}$

$\boxed{\frac{p}{u} = Z_A}$ $\boxed{p = Z . u}$ $Z = \sqrt{\frac{P (R_0)}{X}}$

---

pression acoustique $p(t, M)$
vitesse de fluide $\vec{v}(t, M)$

$p = z . u$

$\frac{p}{u} = z$

$z = \sqrt{\frac{P}{X}}$

$z_{air} = 410 \; Pa / m.s^{-1}$

Intensité d'une onde acoustique

I = puissance de l'onde par unité de surface

puissance $= S . I$ $\boxed{I = p.u}$

puissance $= f . u = p . S . u = S \underbrace{(p u)}_{I} = S I$ $f = P . S$

$\boxed{09/09/2015}$

$$v = \frac{p}{Z} \qquad I = p \cdot \frac{p}{Z} = \frac{p^2}{Z} \qquad \boxed{I = \frac{p^2}{Z}}$$

Niveau Acoustique (Sound Level SL)

Sensation de puissance Acoustique $\sim \log$

I_{ref} = intensité acoustique de référence
$= 1\,pW/m^2$

$$\boxed{SL \text{ en dB} = 10 \log\left(\frac{I}{I_{ref}}\right)}$$

exemple

$$I = 10\ mW/m^2$$

$$SL = 10 \log\left(\frac{10 \cdot 10^{-3}}{10^{-12}}\right)$$

$$= 10 \log\left(\frac{10^{-2}}{10^{-12}}\right)$$

$$= 10 \log\left(10^{-2} \cdot 10^{12}\right)$$

$$= 10 \log\left(10^{10}\right) = 10 \times 10 \log(10)$$

$$= 100 \times 1$$

$$= 100\ dB$$

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$I = 10\ mW/m^2$ | $p = Z.v$ | $Z = 410\ Pa/m.s^{-1}$

$I = Z.v^2$

$v^2 = \frac{I}{Z}$ $\quad v = \sqrt{\frac{I}{Z}}$ $\quad v = \sqrt{\frac{10.10^{-3}}{410}} = 0{,}0049\ m/s = 4{,}9\ m/s$

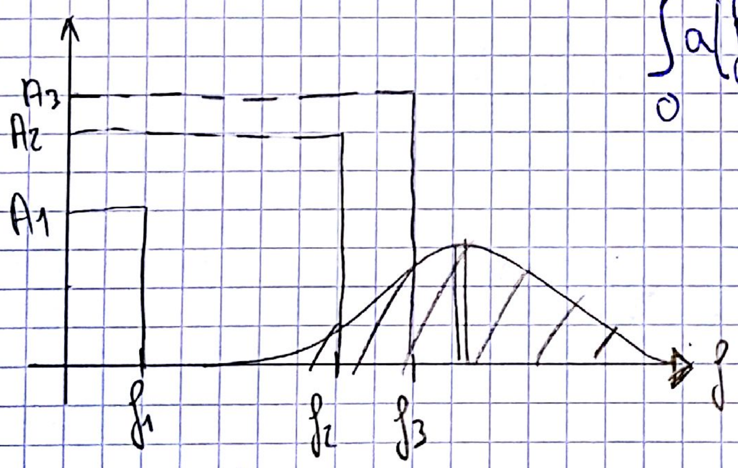
LE COURS COMMENCE

Traitement du signal

1. Analyse spectrale (suite)

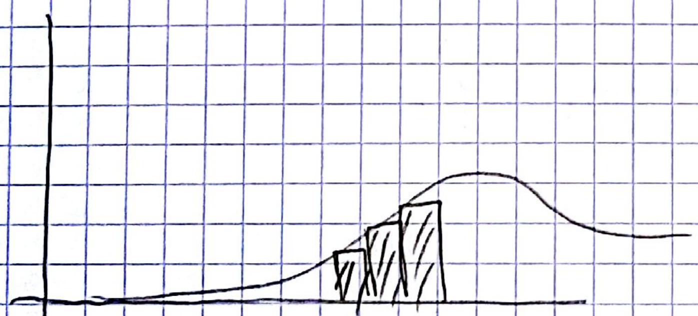
$$x(t) = \sum_{n=0}^{N} A_n \cdot \cos(2\pi f_n t + \varphi_n)$$

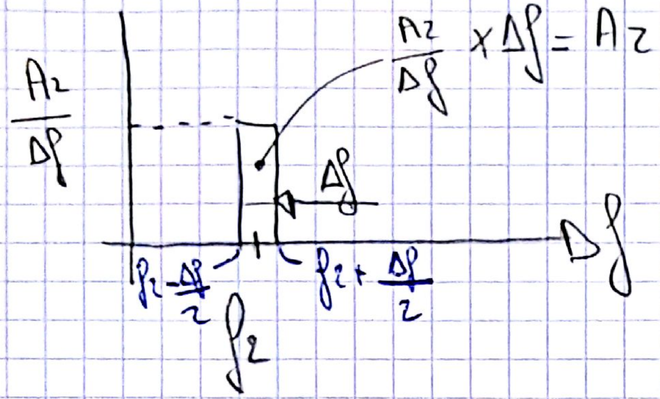
$$\int_0^{+\infty} a(f(t)) \cdot \cos(2\pi f t + \varphi(t))\, df$$



discret   non harmonique

$$\sum_{n=0}^{+\infty} a(n\Delta f) \cdot \Delta f \cdot \cos(2\pi \cancel{f}\, n \cdot \Delta f)$$





$$c_2(t) = A_2 \cdot \cos(2\pi f_2 \cdot t + \varphi_2)$$

$$c_2(t) \approx \int_0^{+\infty} \Pi_{\Delta f}(f - f_2) \cdot A_2 \cos(2\pi f_2 t + \varphi_2)\, df$$

$$\approx \cos(2\pi f t + f_2) \underbrace{\int_0^{+\infty} A_2 \Pi_{\Delta f}(f - f_2)\, df}_{A_2 \Delta f}$$

$$\approx \int_{f_2 - \frac{\Delta f}{2}}^{f_2 + \frac{\Delta f}{2}}$$

$$\approx \lim_{\Delta f \to 0} \int_0^{+\infty} \frac{1}{\Delta f} \Pi_{\Delta f}(f - f_2) \cdot A_2$$

$$\lim_{\Delta f \to 0} \left[\frac{\Pi_{\Delta f}(f_2 - \Delta f)}{\Delta f} \right] = \delta(f - f_2)$$

$$x(t) = \int_{-\infty}^{+\infty} X(f) \cdot e^{i2\pi f t}\, df$$

$$= e^{i2\pi} = \cos(2\pi f t) + i \sin(2\pi f t)$$

$$X(f) = |X(f)| \cdot e^{i\varphi(f)}$$

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$$x(t) = \int_{-\infty}^{+\infty} \boxed{|X(f)| \cdot e^{i\varphi(t)} \cdot e^{i2\pi f \cdot t}} \, df$$

$$\varphi(t) = X(f)$$

$$x(t) = \int_{-\infty}^{+\infty} |X(f)| \cdot e^{i(2\pi f t + \varphi(t))} \cdot df$$

$$e^{i(2\pi f t + \varphi(t))} = \cos(2\pi f t + \varphi(t)) + i\sin(2\pi f t + \varphi(t))$$

$$x(t) = \boxed{\int_{-\infty}^{+\infty} |X(t)| \cdot \cos(2\pi f t + \varphi(t)) \, df}$$

$$+ \int_{-\infty}^{+\infty} i \cdot |X(f(t))| \cdot \sin(2\pi f \cdot t + \varphi(t)) \cdot df$$

$$|X(-f)| = |X(f)|$$

$$\varphi(-f) = -\varphi(f)$$

$$\int_{-\infty}^{+\infty} |X(f)| \cdot \cos(2\pi f \cdot t + \varphi(t)) \, df$$

$$= \int_{0}^{+\infty} |X(f)| \cdot \cos(2\pi f \cdot t + \varphi(t)) \, df$$

$$+ \boxed{\int_{-\infty}^{0} |X(f)| \cdot \cos(2\pi f \cdot t + \varphi(t)) \cdot df}$$

$$\int_{-\infty}^{+\infty} |X(f)| \cos(2\pi f \cdot t + \varphi(t)) \cdot df$$

$$= 2 \int_{0}^{+\infty} |X(f)| \cdot \cos(2\pi f \cdot t + \varphi(t)) \cdot df$$

$$= \int_0^{+\infty} a(t) \cos\left(2\pi . f . t + \varphi(t)\right) . df$$

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Exercice :

① Calculer le spectre de $x(t) = A.\sin(2\pi f_1.t + \varphi_1)$

② En déduire la densité d'amplitude spectrale $a(f)$

$$\boxed{\varphi(f) = \angle X(f)}$$

$$X(f) = \frac{1}{2}\, a(f).\, e^{i\varphi(f)} \qquad \in \mathbb{C}$$

$$a(f) = 2\,|X(f)| \qquad \in \mathbb{R}^+$$

$$\varphi(f) = \angle X(f) \qquad \in \mathbb{R}$$

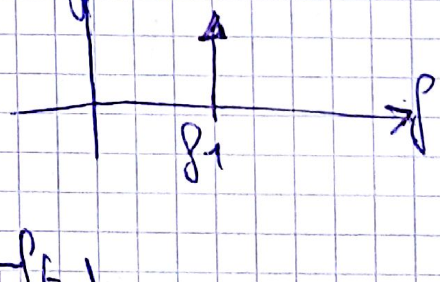
$$TF\left[e^{2i\pi f_2 t}\right] = ?$$

$$TF[\sigma(t)] = \int_{-\infty}^{+\infty} \sigma(t).\, e^{2i\pi f t}\, dt$$

$$= \int_{-\infty}^{+\infty} \delta(t).\, e^{2i\pi f 0}\, d.t$$

$$= e^{2i\pi f 0} \int_{-\infty}^{+\infty} \delta(t)\, dt$$

$df = 1$



$$TF^{-1}[\delta(f)] = \int_{-\infty}^{+\infty} \delta(f).\, e^{2i\pi f.t}\, df = 1$$

$$TF^{-1}[\delta(f)] = 1$$

$$TF^{-1}[\cancel{X}\ \delta(f - f_1)] = \int_{-\infty}^{+\infty} \delta(f - f_1).\, e^{2i\pi f t}\, dt$$

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$$= e^{2i\pi f_1 . t} \int_{-\infty}^{+\infty} \delta(f_1 - f_1)\, df = e^{2i\pi f_1 . t}$$

$$TF[e^{2i\pi f_1 t}] = \delta(f - f_1)$$

$$= \frac{e^{i(2\pi f_1 t + \varphi_1)} - e^{-i(2\pi f_1 t - \varphi_2)}}{2i}$$

$$TF[\sin(2\pi f_1 . t + \varphi_1)]$$

$$= TF\left[\frac{e^{i(2\pi f_1 t + \varphi_1)} - e^{-i(2\pi f_1 t + \varphi_1)}}{2i}\right]$$

$$= \frac{1}{2i}\left[TF[e^{i(2\pi f_1 t) + \varphi_1}] - TF[e^{-i(2\pi f_1 t + \varphi_1}]\right]$$

$$= \frac{1}{2i}\left[TF[- e\right.$$

$$= \frac{1}{2i}\left[e^{i\varphi_1}\, TF[e^{i2\pi f_1 t}] - e^{-i\varphi_1}\, TF[e^{-i2\pi f_1 t}]\right]$$

Dirac

$$\boxed{X(f) = \underbrace{\frac{e^{i\varphi_1}}{2i}\, \delta[f - f_1]}_{} - \underbrace{\frac{e^{-i\varphi_1}}{2i}\, \delta[f + f_1]}_{}}$$

$$|X(f)| = | \quad | + | \quad |$$

$$= \left|\frac{e^{i\varphi_1}}{2e^{i\frac{\pi}{2}}}\, \delta(f - f_1)\right| + \left|-\frac{e^{-i\varphi_1}}{2e^{i\pi/2}}\, \delta(f + f_1)\right|$$

$$= \left|\frac{A}{2}\right| \left|e^{i(\varphi_1 - \frac{\pi}{2})}\right| \delta(f - f_1) + \frac{A}{2}\left|e^{i(-\varphi_1 - \frac{\pi}{2})}\right| \delta(f + f_1)$$

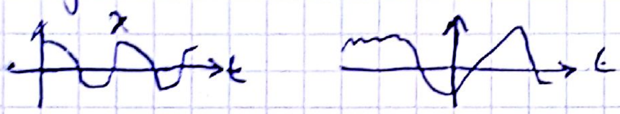
$$= A\, \frac{1}{2}\left[\delta(f - f_1) + \delta(f + f_1)\right]$$

TAN - $\boxed{17/09/15}$

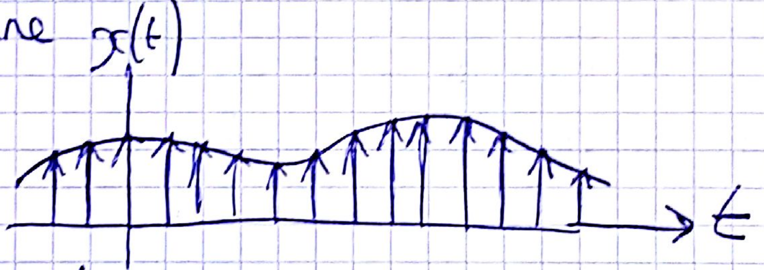
$$x(t) \longrightarrow \boxed{\mathcal{H}} \longrightarrow y(t)$$

fonction $\underset{\in \mathbb{R}}{x} \xmapsto{f} \underset{\in \mathbb{R}}{y}$

opérateur fonction $\xrightarrow{\mathcal{H}}$ fonction



$\mathcal{H}$ $\begin{cases} \text{- linéaire} \\ \text{- causal} \\ \text{- stationnaire} \end{cases}$



$$y(t) = \mathcal{H}[x(t)]$$

$$\sum_{m=-\infty}^{+\infty} x(m\Delta t) . \delta(t - m\Delta t)$$

$$= \int_{-\infty}^{+\infty} x(\tau) . \delta(t-\tau) \, d\tau$$

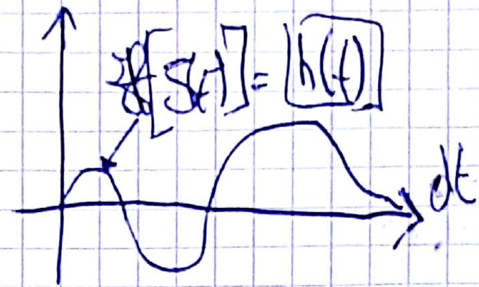
$$= \int_{-\infty}^{+\infty} x(t) . \delta(t-\tau) \, d\tau$$

$$= x(t) . \int_{-\infty}^{+\infty} \delta(t-\tau) \, d\tau$$

$$= x(t) . 1 = \boxed{x(t)}$$

$$y(t) = \int_{-\infty}^{+\infty} x(\tau) \cdot h(t-\tau)\, d\tau$$

$$y(t) = x(t) * h(t)$$



$$\mathcal{H}[\cos(2\pi f_0 t + \varphi)] = e^{i\varphi}$$

$$\frac{e^{i\varphi}}{2} \underbrace{\mathcal{H}\left[e^{2i\pi f_0 t}\right]}_{\mathcal{H}[e^{2\pi f}} + \frac{e^{i\varphi}}{2} \mathcal{H}\left[e^{-2\pi f_0 t}\right]$$

$$\mathcal{H}\left[e^{2i\pi f_0 t}\right] = \int_{-\infty}^{+\infty} e^{-2i\pi f_0 t \tau} \; h(t-\tau)\, d\tau$$

$$t' = t - \tau \longrightarrow \tau = t - t'$$

$$\mathcal{H}\left[e^{2i\pi f_0 t}\right] = \int_{-\infty}^{+\infty} - e^{-2i\pi f_0 (t-t')} \cdot h(t')\, dt'$$

$h(t)$ réponse impulsionnelle

$h(t) = \mathcal{H}[\delta(t)]$

$H(f)$ réponse fréquentielle

$H(f) = TF[h(t)](f)$

$$= \int_{\infty} - e^{-2i\pi f_0 (t-t')} \; h(t') \cdot dt'$$

$$= \cancel{-}\int_{-\infty}^{+\infty} e^{-2i\pi f_0 (t-t')} h(t')\, dt'$$

$$= \int_{-\infty}^{+\infty} e^{2\pi f_0 t} \; e^{-i 2\pi f_0 t'} \; h(t')\, dt'$$

$$= e^{2i\pi f_0 t} \boxed{\int_{-\infty}^{\infty} e^{-i 2\pi f_0 t'} \; h(t')\, dt'}$$

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$$x(t) \longrightarrow \boxed{H} \longrightarrow y(t)$$

$$y(t) = x(t) * h(t)$$

$$TF[y(t)] = TF[x(t) * h(t)]$$

$$= TF[x(t)] \cdot TF[h(t)]$$

$$= X(f) \cdot H(f)$$

~~réponse~~ fonction de transfert

$$\hat{H}(s) = TL[h(t)] = \int_0^{+\infty} h(t) \cdot e^{\boxed{-st}} \cdot dt$$

$$H(f) = TF[h(f)] = \int_0^{+\infty} h(t) \, e^{\boxed{-2\pi i f t}} \, dt$$

$$s = \sigma + 2i\pi f$$

$$\hat{H}(2i\pi f) = H(f)$$

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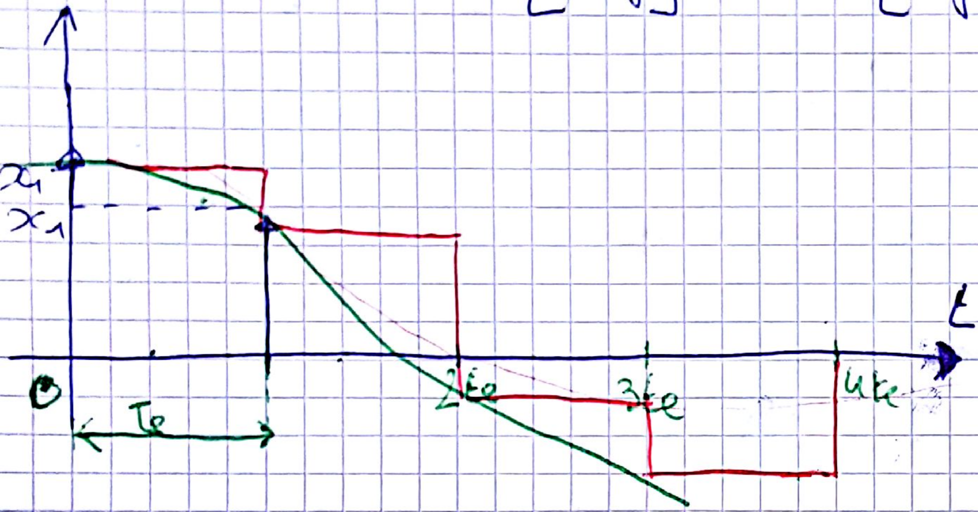
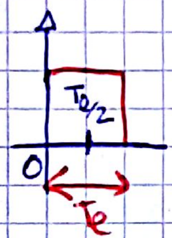
TAN - $\boxed{12/10/15}$

$$\overset{\circ}{X}(f) = TF[\overset{\circ}{x}(t)]$$

$$X(f) = \overset{\circ}{X}(f) \times \Pi_{f_e}(f)$$

$$x(t) = TF^{-1}[X(f)] = TF^{-1}[\overset{\circ}{X}(f) \times \Pi_{f_e}(f)]$$





$$\delta(t) \rightarrow \Pi_{T_e}\left(t - \frac{T_e}{2}\right)$$

$$\delta(t - \theta) \rightarrow \Pi_{T_e}\left(t - \theta - \frac{T_e}{2}\right)$$

$$\bar{x}(t) = \Pi_{T_e}(t) * \overset{\circ}{x}(t) * \delta\left(t - \frac{T_e}{2}\right)]$$

$$TF[\bar{x}(t)] = TF\left[\Pi_{T_E}(t) * \overset{\circ}{x}(t) * \delta\left(t - \frac{T_e}{2}\right)\right]$$

$$= TF[\Pi_{T_E}(t)] \cdot TF[\overset{\circ}{x}(t)] \cdot TF\left[\delta\left(t - \frac{T_e}{2}\right)\right]$$

$$TF[\overset{\circ}{x}(t)] = \underbrace{TF[x(t)]}_{X(f)} \cdot TF[Ш_{T_e}(t)]$$

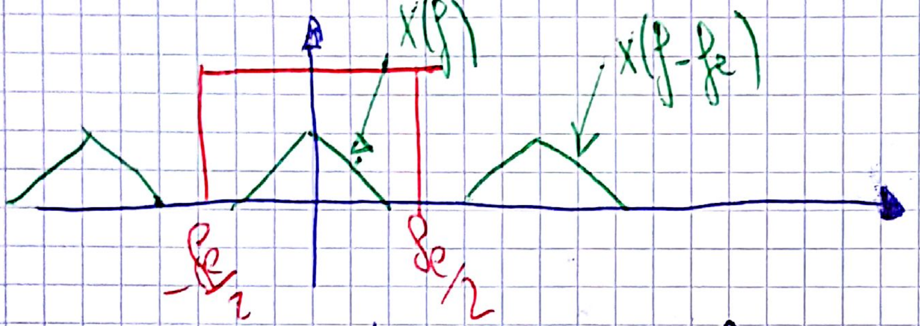
$$= X(f) * f_e Ш_{f_e}(f)$$

$$= X(f) * f_e \sum_{-\infty}^{+\infty} \delta(f - m f_e)$$

$$= f_e \sum_{m=-\infty}^{+\infty} X(f) * \delta(f - m f_e)$$

$$= f_e \sum_{m=-\infty}^{\infty} X(f - m f_e)$$

$$f(t) * \delta(t - \tau) = f(t - \tau)$$



$$TF[\Pi_{T_e}(t)] = \int_{-\infty}^{+\infty} \Pi_{T_e}(t)\, e^{-2i\pi f t}$$

$$TF[\Pi_{T_e}(t)] = \left[\frac{e^{-2\pi f t}}{-2i\pi f}\right]_{-\frac{T_e}{2}}^{\frac{T_e}{2}} = \frac{e^{-\cancel{2}i\pi f \frac{T_e}{\cancel{2}}}}{-2i\pi f} - \frac{e^{-2i\pi f \cdot \frac{-T_e}{2}}}{-2i\pi f}$$

$$= \frac{e^{-i\pi f T_e} - e^{+i\pi f T_e}}{-2i\pi f} = \frac{e^{i\pi f T_e}}{2i\pi}$$

$$TF[\Pi_{T_e}(t)] = \frac{T_e \sin(\pi f T_e)}{\pi f T_e}$$

$$= T_e \cdot \operatorname{sinc}(\pi f T_e)$$

sinus cardinal

Scanned by CamScanner

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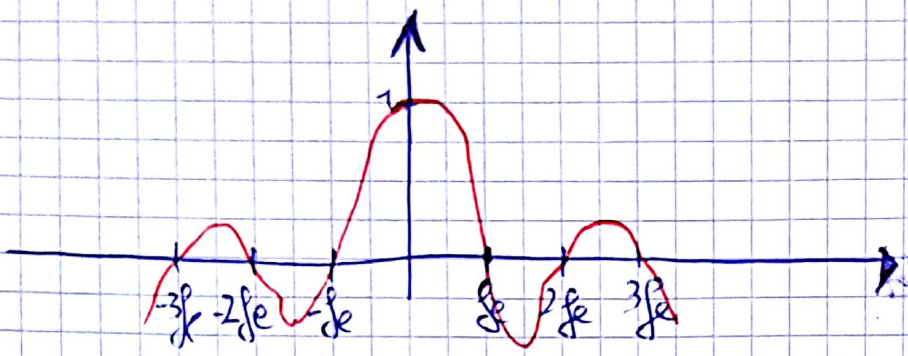
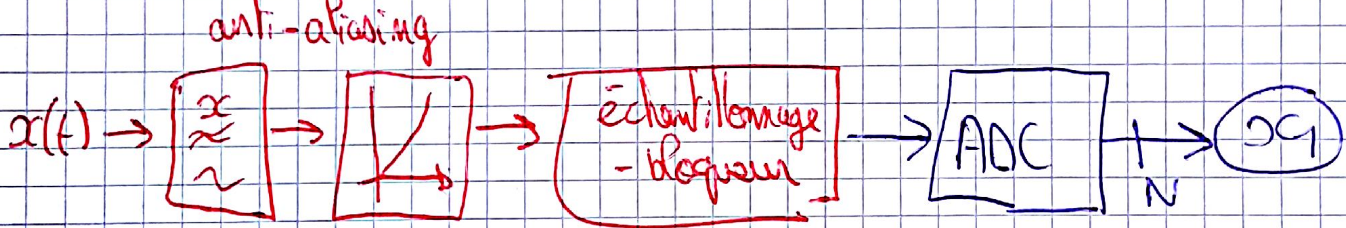


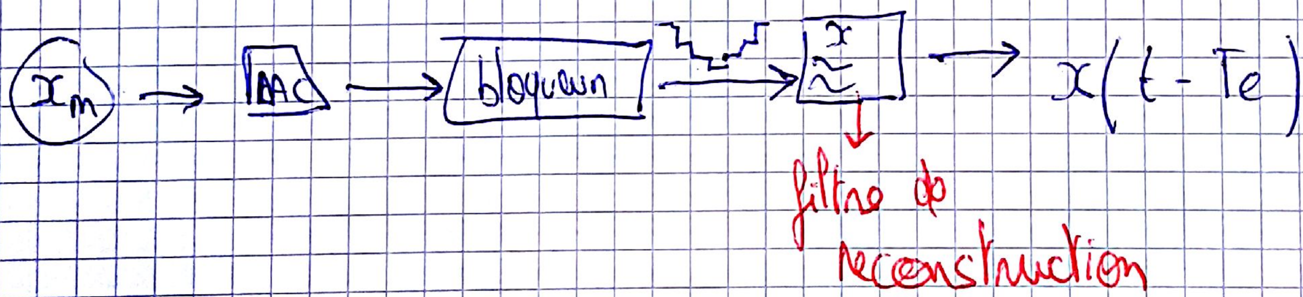
$$\Rightarrow \mathrm{sinc}\left(\pi \frac{f}{f_e}\right) = \frac{\sin\left(\pi \frac{f}{f_e}\right)}{\pi \frac{f}{f_e}} \underset{f \approx 0}{\approx} \frac{\pi \frac{f}{f_e}}{\pi \frac{f}{f_e}} = \boxed{1}$$

$$TF\left[x^*(t)\right] = TF\left[x(t)\right] * TF\left[Ш_{Te}(t)\right]$$

$$= f_e \sum_{n=-\infty}^{+\infty} X(f - n f_e)$$

$$= f_e \times \text{Périodisation}_{f_e}\left[X(f)\right]$$





09/11/15 - TAN

CONVOLUTION NUMÉRIQUE

1. Rappel : convolution analogique

$$x(t) \longrightarrow \boxed{\mathcal{H}} \longrightarrow y(t)$$

$$y(t) = x(t) * h(t)$$

avec $h(t) = \mathcal{H}[\delta(t)]$

2. Convolution numérique

$$\begin{cases} y_n = y(nTe) \\ x_n = x(nTe) \\ h_n = h(nTe) \end{cases}$$

que vaut y_n relativement à x_n et h_n ?

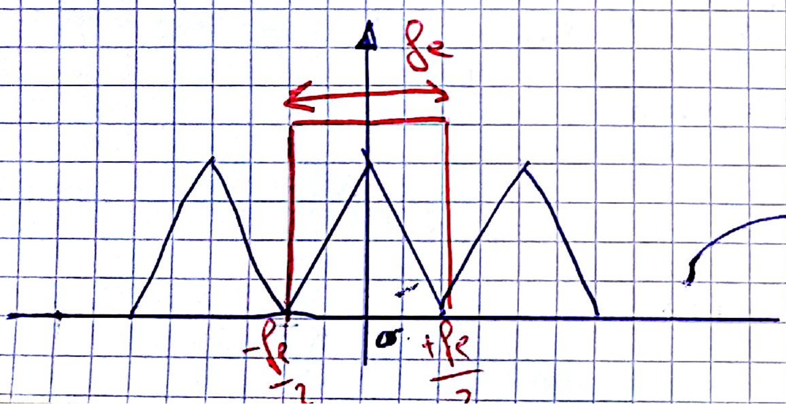
$$\overset{\circ}{x}(t) = \sum_{n=-\infty}^{+\infty} x_n \cdot \delta(t - nTe) = \overset{\circ}{x}(t)$$

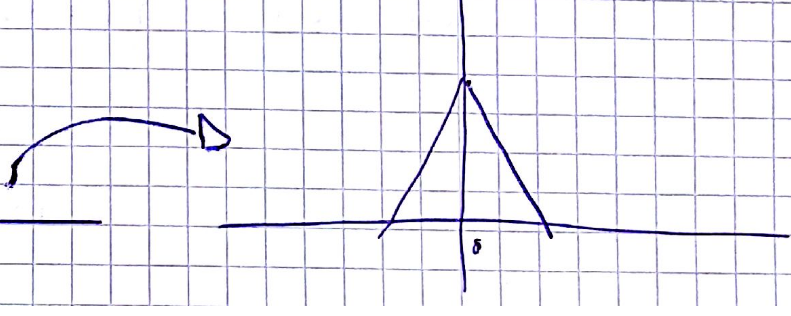
signal échantillonné

$x(t) = g(t) * \overset{\circ}{x}(t)$ avec $g(t) = K \cdot TF^{-1}[\Pi_{fe}(f)]$

$\Pi_{fe}(f) = 1$ pour $f \in \left[-\frac{fe}{2}, \frac{fe}{2}\right]$

$= 0$ sinon





$$h'(t) = \sum_{-\infty}^{+\infty} h_m \, \delta(t - mTe)$$

$$h(t) = g(t) * h'(t)$$

On remplace : $y(t) = g(x)^{* x'(t)} * g(t) * h'(t)$

$$= \sum_{-\infty}^{+\infty} x_m \, \delta(t - mTe) * \sum_{-\infty}^{+\infty} h_q \, \delta(t - qTe) * g(t) * g(t)$$

$$TF[g(t) * g(t)] = TF[g(t)] \times TF[g(t)]$$

$$= \left(K . \Pi_{fe}(f)\right)^2$$

$$= K^2 . \Pi_{fe}^{2}(f)$$ nom de la porte

$$g(t) * g(t) = TF^{-1}\left[TF[g(t) * g(t)]\right]$$

$$= TF^{-1}\left[K^2 \Pi_{fe}(f)\right]$$

$$= K^2 \, TF^{-1}\left[\Pi_{fe}(f)\right]$$

$$= K^2 \frac{g(t)}{\cancel{K}} = \boxed{K . g(t)}$$

$$y(t) = \sum_{\infty}^{\infty} x_m . \delta(t - mTe) * \sum_{\infty}^{\infty} h_q . \delta(t - qTe) * K . g(t)$$

$$= K_{g(t)} * \sum \sum x_m . \delta(t - mTe) * h_q . \delta(t - qTe)$$

$$= \sum \sum x_m h_q \, \delta(t - mTe - qTe) * K . g(t)$$

$$= \sum \sum x_m h_q \, \delta(t - (m+q)Te) * K . g(t)$$

$$n = m + q \Rightarrow q = n - m$$

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$$y(t) = \sum_{-\infty}^{\infty} \underbrace{\boxed{\sum_{-\infty}^{+\infty} x_m h_{n-m}}}_{y'_n} \delta(t - nTe) * k \cdot g(t)$$

$$y(t) = \sum_{-\infty}^{\infty} y'_n \cdot \delta(t - nTe) * k\, g(t)$$

$$y(t) = \sum y_n \cdot \delta(t - nTe) * g(t)$$

$$k\, y'_n = y_n$$

$$y_n = \boxed{k \sum x_m h_{n-m}} \qquad \boxed{y_n = k \sum x_m \cdot h_{n-m}}$$

$$k = Te \qquad y_n = Te \sum x_{n-m} \cdot h_m$$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(t-\theta)\, h(\theta)\, d\theta$$

$$\boxed{y_n = Te \cdot \sum_{-\infty}^{+\infty} x_{n-m} \cdot h_m}$$

$h(t)$ dure 10s

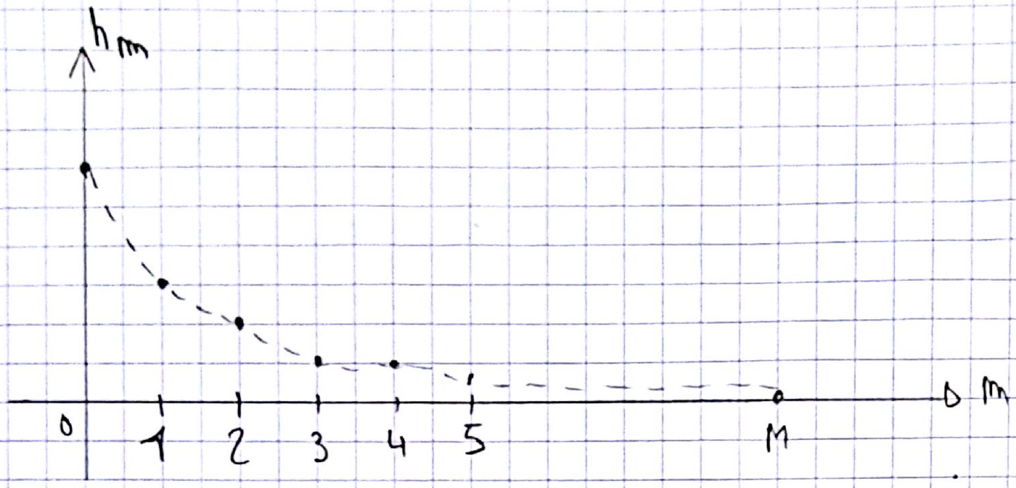
$h(t)$ causal

$$\boxed{x_n * h_n = \sum_{m=-\infty}^{\infty} x_{n-m} \cdot h_m}$$

$$\boxed{y_n = Te \cdot x_n * h_n}$$

1) donner les bornes pour m

2) calculer le nombre d'additions et de multiplications requises pour calculer chaque échantillon y_n



$M.Te = T = 10s$

$Te = \dfrac{1}{fe} \qquad fe = 22{,}68.10^{-6}s$

$\cancel{M = \dfrac{1}{Te} = 10s}$

$M = \dfrac{10}{22{,}68.10^{-6}} = \dfrac{10^7}{22{,}68}$

$M = 441\ 000$

Machine : 0,1 Gips
4 instructions / boucle

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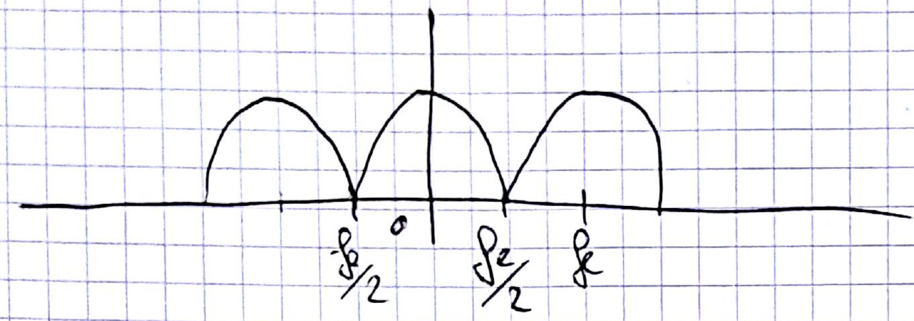
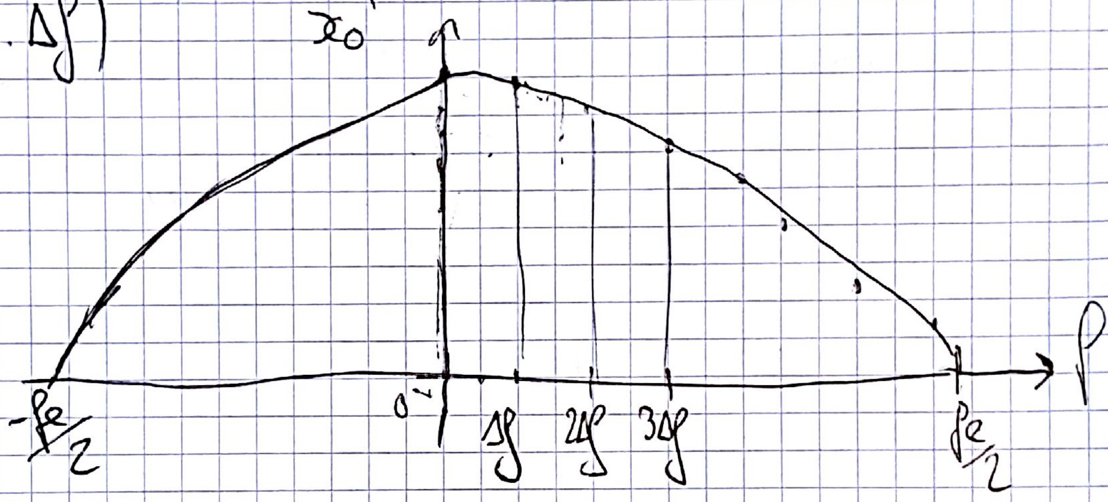


On va échantillonner le spectre $X(f)$ avec un pas Δf

$$X'_k = X(k.\Delta f)$$



$$X'_k = X(k.\Delta f) = a.X(k.\Delta f)$$

$$x(t) = \sum_{n=-\infty}^{+\infty} TF\left[\sum_{n=-\infty}^{+\infty} x_n . \delta(t-nTe)\right]$$

$$= \sum_{n=-\infty}^{+\infty} Tf\left[x_n . \delta(t-nTe)\right]$$

$$= \sum_{-\infty}^{+\infty} x_n \, TF\left[\delta(t-nTe)\right]$$

$$= \sum_{-\infty}^{+\infty} x_n \underbrace{TF\left[\delta(t)\right]}_{1} . e^{-2i\pi n Te . f}$$

$$= \sum x_n . e^{-2i\pi nTe f} = \sum x_n . e^{-2i\pi nTe k\Delta f}$$

$n < 0 \Rightarrow x_n = 0$

$n > (N-1) \Rightarrow x_n = 0$

$$\mathring{X}(k\Delta f) = \sum x_n \cdot e^{-2i\pi nk \cdot Te \Delta f}$$

$$\boxed{\mathring{X}(f) = \sum_{n=0}^{N-1} x_n \cdot e^{-2i\pi n \cdot Te \cdot f}}$$

$X(f) = a\,\mathring{X}(f)$ pour $f \in \left[\frac{-fe}{2} ; \frac{fe}{2}\right]$

On va appliquer le théorème de l'échantillonnage

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DFT → FFT

1. Introduction

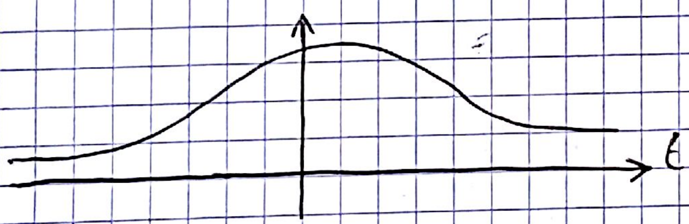
Nous avions vu la TF

- Quel est l'équivalent numérique ? (avec des signaux échantillonnés)
  > DFT
- Peut-on la calculer plus vite ?
  > FFT

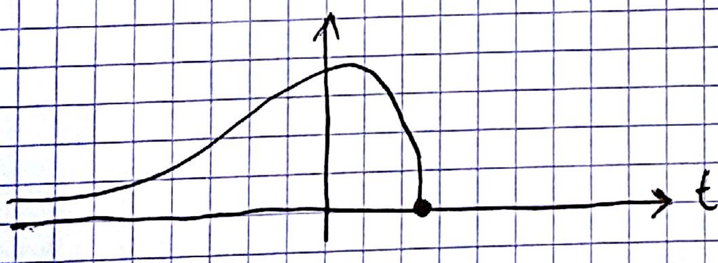
2. Echantillonnage de la TF

$x(t)$ $\quad x_n = x(nTe)$

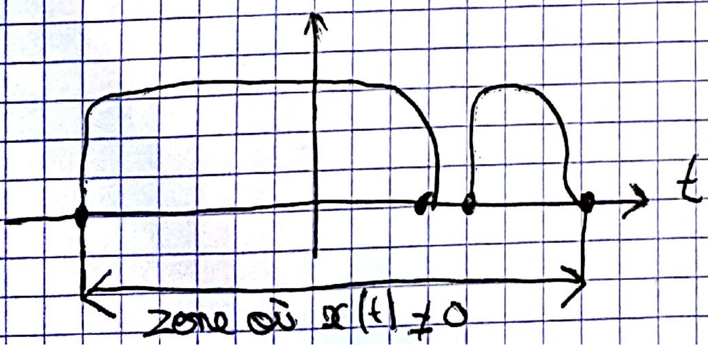
on suppose que $x(t)$ est de support borné



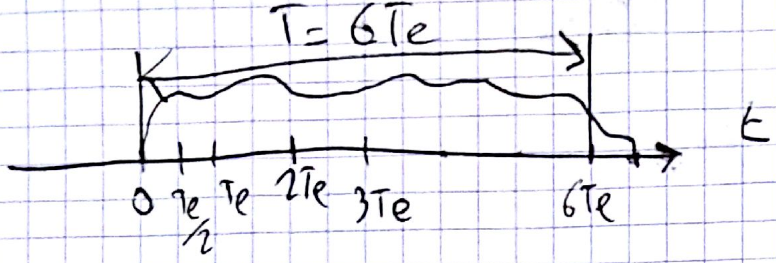
NON, pas borné



NON



OUI

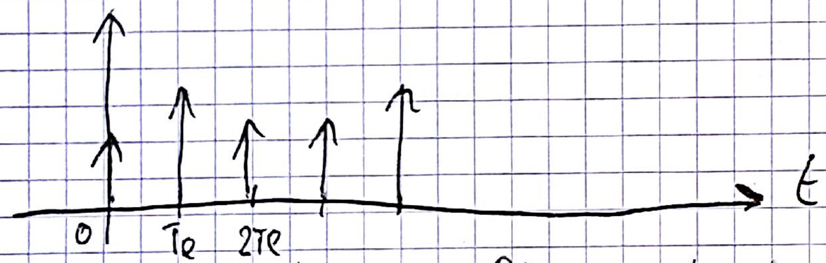


$\underbrace{x_0 = x_5}_{N=6 \text{ échantillons}}$ $\qquad x_0$ à x_{n-1}

Spectre de $x(t) = X(f) = TF[x(t)]$

$$= \int_{-\infty}^{+\infty} x(t)\, e^{-2i\pi . f . t} \, dt$$

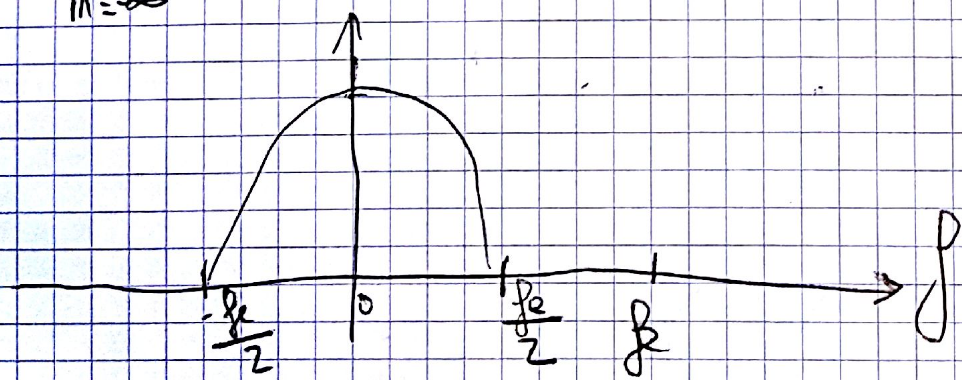
<u>Signal échantillonné</u> $= \dot{x}(t) = \sum_{n=-\infty}^{+\infty} x_n . \delta(t - nTe)$



spectre de $\dot{x}(t) = \dot{X}(f) = TF[\dot{x}(t)]$

= a spectre de $X(f)$ périodisé

$= a \sum_{n=\infty}^{+\infty} X(f - nfe)$



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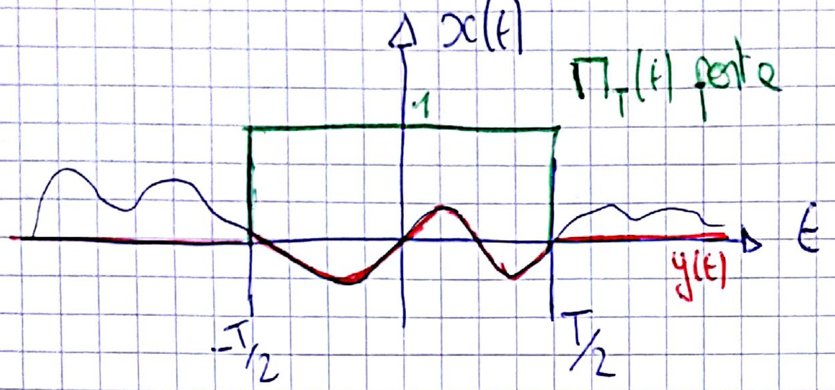
FENÊTRAGE

1 - Introduction



$$X(f) = TF[x(t)] = \int_{-\infty}^{+\infty} x(t) \cdot e^{-2i\pi f t} dt$$

$$\hat{X}(f) = TF[y(t)]$$ approximation

$$y(t) = x(t) \times \Pi_T(t))$$

$$\hat{X}(f) = TF[x(t) \cdot \Pi_T(t)]$$

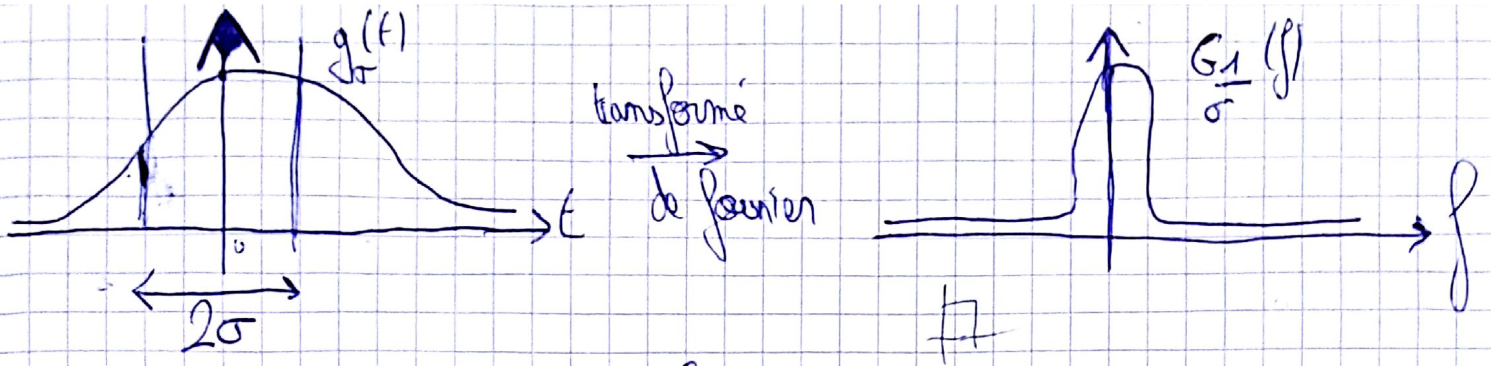
$$= TF[x(t)] * TF[\Pi_T(t)]$$

$$= X(f) * \underbrace{TF[\Pi_T(t)]}_{W(f)}$$

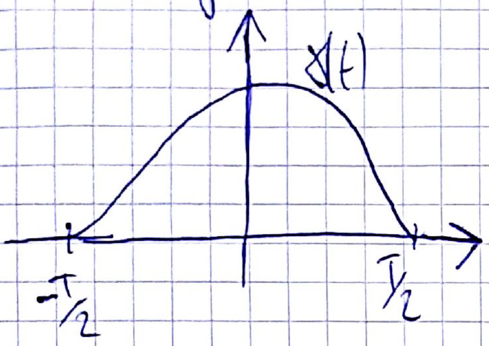
que doit valoir $W(f)$ pour que $\hat{X}(f) \simeq X(f)$

$$= X(f) * \delta(f) = X(f)$$

Au lieu d'utiliser la fonction porte, on utilise une gaussienne



D'autres fonctions peuvent être utilisées :



$$\gamma(t) = \left(\left(\cos\left(2\pi\frac{t}{T}\right) + 1\right) \backslash\text{div } 2\right) * \Pi_T(t)$$

fonction de Hann (Solo)

TF à court terme et Analyse temps - fréquence

1- Introduction :

$F_{0♂} = 100\ Hz \quad T_{0♂} \approx 10ms$

$F_{0♀} = 200Hz \quad T_{0♀} \approx 5ms$

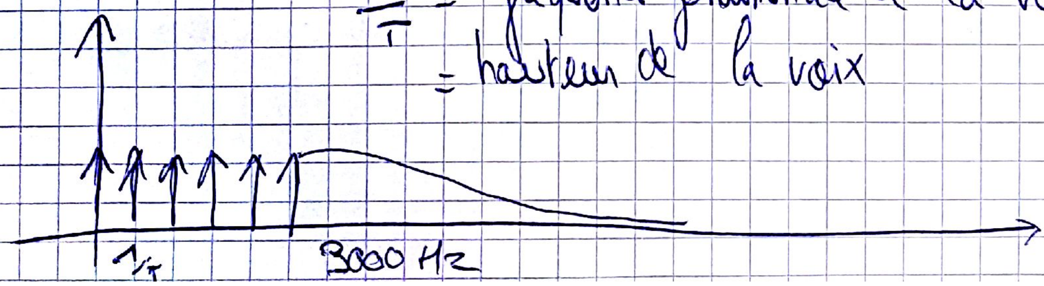
$v(t) = e(t) * h(t)$ la parole

$$\begin{aligned} v(f) &= TF[v(t)] \\ &= TF[e(t) * h(t)] \\ &= TF[e(t)] \times TF[h(t)] \\ &= E(f) \times H(f) \end{aligned}$$

excitation

réponse fréquentielle

$\frac{1}{T}$ = fréquence fondamentale de la voix
= hauteur de la voix



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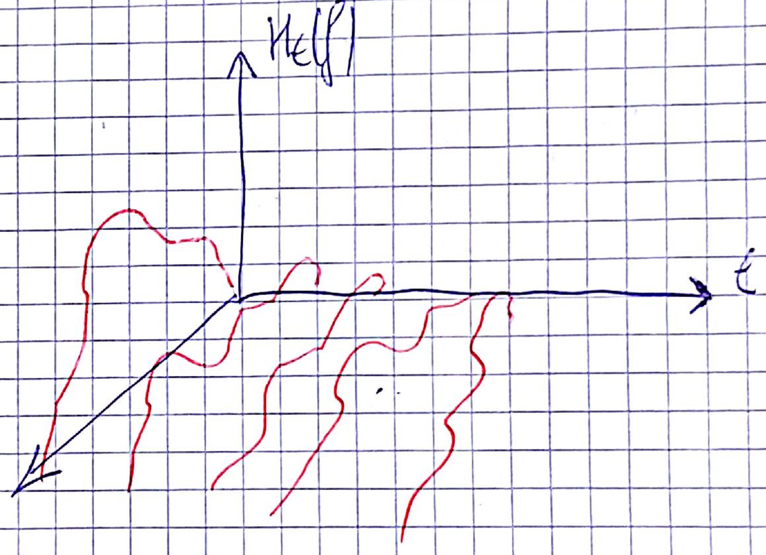
$$X_t(f) = \text{STFF}_t\left[x(\tau)\right]$$
$$= \text{TF}\left[x(\tau) \times g(\tau - t)\right]$$
$$= \int_{-\infty}^{+\infty} x(\tau) \times g(\tau - t) \cdot e^{-2i\pi f \cdot \tau} \cdot d\tau$$
$$= \int_{t-T/2}^{t+T/2} x(\tau) \times g(\tau - t) \cdot e^{-2i\pi f \cdot \tau} \cdot d\tau$$

$$V_t(f) = E_t(f) \times H_t(f)$$

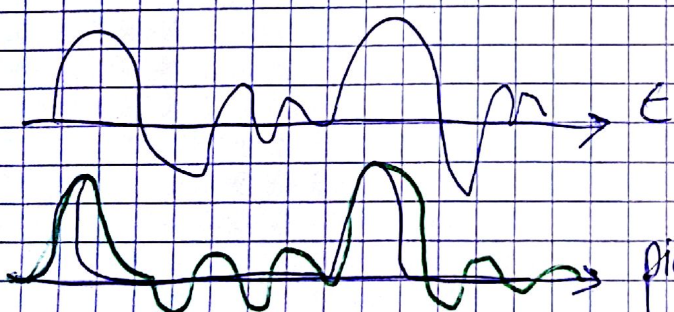
$$y_t(\tau) = x(\tau) \times g(\tau - t)$$

$$y_{t+\Delta t}(\tau) = x(\tau) \times g(\tau - t - \Delta t)$$



$\Delta t = \frac{T}{2}$ = période d'échantillonnage

T = durée de la fenêtre = durée du morceau de signal



$T \gg$ période du signal fondamental $\approx 10\,ms$

$\Delta t = \dfrac{T}{2} \ll$ temps de changement du spectre

$$0{,}01\,s \ll T \ll \frac{1}{2}\,s$$

$$T \approx \frac{1}{10}\,s$$

N échantillons

$f_e = 44\,100\,Hz$ (CD)

$= f_e \cdot T \approx \underbrace{4410}_{4096}$ (puissance de 2 la plus proche)

$$T = \frac{N}{f_e} = \frac{4096}{44100} = 0{,}09288798\,s$$