

19/20

GARNIER

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Prépa 1A

Intens. Math.

16/03/15

Exercice 1.

$$\begin{aligned} \circ P(A \cap B) &= P(A) + P(B) - P(A \cup B) = \frac{2}{3} + \frac{1}{5} - \frac{4}{5} \\ &= \frac{10}{15} + \frac{3}{15} - \frac{12}{15} = \frac{1}{15} \end{aligned}$$

$$\circ P(A \cap \bar{B})$$

$$P(\bar{B}) = 1 - P(B) = \frac{4}{5}$$

$$P(A) - P(A \cap B) = \frac{2}{3} - \frac{1}{15} = \frac{9}{15}$$

~~$$P(A \cap \bar{B}) = \frac{2}{3} + \frac{4}{5} - \frac{4}{5} = \frac{2}{3}$$~~

$$\begin{aligned} \circ P(A \cup \bar{B}) &= P(A) + P(\bar{B}) - P(A \cap \bar{B}) \\ &= \frac{2}{3} + \frac{4}{5} - \frac{2}{3} = \frac{4}{5} = \frac{12}{15} \end{aligned}$$

$$\circ P(\bar{A} \cap \bar{B})$$

$$P(\bar{A}) = \frac{1}{3}$$

$$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B) = \frac{1}{5}$$

$$\circ P(\bar{A} \cap B) = P(B) - P(A \cap B) = \frac{1}{5} - \frac{1}{15} = \frac{2}{15}$$

$$P(A \Delta B) = P(A \cup B) - P(A \cap B) = \frac{4}{5} - \frac{1}{15} = \frac{11}{15}$$

Exercice 2.

$$1. \sum_{k=1}^6 c \cdot k = 1$$

$$\Leftrightarrow 21 \cdot c = 1$$

$$\Leftrightarrow c = \frac{1}{21} \quad /$$

$$2. P(A) = \frac{1}{21} + \frac{2}{21} = \frac{3}{21} = \frac{1}{7} \quad /$$

$$P(B) = \frac{1+6}{21} = \frac{7}{21} = \frac{1}{3} \quad /$$

$$P(A \cap B) = P(\{1\}) = \frac{1}{21} = P(A) \times P(B) \quad /$$

Donc les deux événements sont indépendants. /

$$3. P_{A \cup B}(A) = \frac{P(A \cap (A \cup B))}{P(A \cup B)}$$

$$P(A \cup B) = P(\{1, 2, 6\}) = \frac{9}{21}$$

$$P(A \cap (A \cup B)) = P(A) = \frac{1}{7}$$

$$P_{A \cup B}(A) = \frac{\frac{1}{7}}{\frac{9}{21}} = \frac{1}{7} \cdot \frac{21}{9} = \frac{21}{63} = \frac{7}{21} = \frac{1}{3} \quad /$$

$$P_{A \cup B}(B) = \frac{\frac{7}{3}}{\frac{9}{21}} = \frac{7}{3} \cdot \frac{21}{9} = \frac{21}{2.7} = \frac{7}{9} \quad /$$

Il s ne sont pas indépendants car:

$$P_{A \cup B}(A) \neq P(A) \quad \left(\text{et} \quad P_{A \cup B}(B) \neq P(B) \right)$$

$$4. P_A(A \cup B) = \frac{P((A \cup B) \cap \Omega)}{P(\Omega)} = 1$$

$$\bullet P_A(\bar{A} \cap \bar{B}) = \frac{P((\bar{A} \cap \bar{B}) \cap \Omega)}{P(\Omega)} = 0$$

$$\bullet P_A(A) = 1$$

$$\bullet P_A(B) = \frac{P(B \cap \Omega)}{P(\Omega)} = \frac{P(\{1, 2\})}{P(\Omega)} = \frac{1}{21} \cdot \frac{1}{7}$$

$$= \frac{1}{21} \cdot \frac{7}{7} = \frac{7}{21} = \frac{1}{3}$$

$$\bullet P_A(A \cap B) = \frac{P((A \cap B) \cap \Omega)}{P(\Omega)} = \frac{P(\{1, 2\})}{P(\Omega)}$$

$$= \frac{1}{21} = \frac{1}{3} \cdot \frac{1}{7}$$

Exercice 3.

1. Les événements A_1 sont indépendants donc

$$P(A_1 \cap A_2) = P(A_1) \times P(A_2) \quad \text{ou} \quad P(A_1) \text{ et } P(A_2)$$

ne sont pas nuls, par conséquent les ensembles ne sont pas disjoints et donc pas incompatibles.

TB/
oui
mais il y a
plus simple!

$$2. P(A_1 \cap A_2) = P(A_1) \times P(A_2) = \frac{1}{3} \times \frac{2}{5} \times \frac{1}{3}$$

$$= \frac{2}{27} \cdot \frac{1}{9}$$

$$P(A_1 \cup A_2) = \frac{1}{3} + \frac{2}{5} \times \frac{1}{3} = \frac{1}{3} + \frac{2}{15} = \frac{1}{3} + \frac{2 \cdot 1}{3 \cdot 5} = \frac{1}{3} + \frac{2}{15} = \frac{5}{15} + \frac{2}{15} = \frac{7}{15}$$

$$\frac{2}{9} - \frac{1}{9} = \frac{6}{9} - \frac{1}{9} = \frac{5}{9} \quad /$$
$$= \frac{5}{9} - \frac{2}{27} = \frac{15}{27} - \frac{2}{27} = \frac{13}{27}$$

$$\} P(A_n \cap \dots \cap A_{n+n}) = \left(\frac{2}{3}\right)^{n+n-1} \times \frac{1}{3} - \left(\frac{2}{3}\right)^n$$

Correction: $P(A_n) \cdot P(A_{n+n}) = \left(\frac{1}{3}\right)^{n+1}$